ONE-DIMENSIONAL STABILITY OF NORMAL GAS COMBUSTION

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By analyzing the behavior of one-dimensional perturbations imposed on a normal flame in gases we have established an intensive stability of combustion entirely attributable to the stabilization effect of compressibility.

The hydrodynamic instability of a normal gas flame to relatively small two-dimensional perturbations distorting the discontinuity front of the flame was first noted by L. D. Landau [1] with the assumption that the medium was incompressible. Subsequently, the various aspects of this problem were dealt with in similar manner by other authors (for example, [2–5]). However, the investigation of flame stability with respect to one-dimensional perturbations which do not alter its geometric shape cannot be carried out within the framework of an incompressible medium, since the absence of deformation of the medium totally excludes the possibility of any of the perturbations having any effect.

Later on we will therefore treat the one-dimensional stability of combustion with consideration of the medium's compressibility. The existence of only a single characteristic linear dimension—the width L of the flame zone—requires the mandatory consideration of the internal structure of the front, in contrast to the two-dimensional case in which it would be possible to neglect L, as in [1], as small in comparison with the perturbation wavelength λ along the flame.

Let a steady two-dimensional flame be enclosed between the planes x = −L and x = 0. The gas flowing in the positive direction of the x-axis will then pass successively through three regions occupied by the initial combustible mixture "1" ($x \le -L$), the combustion process "3" ($-L \le x \le 0$), and the product of combustion "2" ($x \ge 0$). The gasdynamic parameters p, ρ, v, c, S, $\varkappa$, and c_p are marked with numerical subscripts for these regions, choosing the constant parameters of region "3" as the average quantities through the width. If the flame is displaced $\varepsilon = B\exp\omega t$ as a result of any random internal factors, it will serve as the source of a perturbed state of the medium in the form of acoustic (p'_{jk}, v'_{jk}) and entropy (S'_j) waves which are described, in analogy with [6], by the following solutions of linearized gasdynamic equations (the Euler equation, the equation of continuity, and the entropy equation):

$$v'_{jk} = A_{jk}\psi_{jk},\quad \frac{p'_{jk}}{\rho_j v_j} = -\left(1+\frac{\omega}{v_{jk}v_j}\right)A_{jk}\psi_{jk},$$

$$S'_j = D_j\psi_{j3},\quad \gamma_{jk} = -\frac{\omega}{v_j}\,\frac{M_j}{M_j+(-1)^k},\quad \gamma_{j3} = -\frac{\omega}{v_j},$$

$$\psi_{jk} = \exp\left[\gamma_{jk}(x+L)+\omega t\right],\quad M_j = \frac{v_j}{c_j}$$

$$(j = 1,\ 2,\ 3;\ k = 1,\ 2). \tag{1}$$

Since the only cause of the perturbations lies within the combustion process, in the regions of the original mixture (j = 1) and in the regions of the products of combustion (j = 2) we should concern ourselves only with the acoustic waves radiated by the flame upstream (k = 1) and downstream (k = 2); however, the entropy wave transported by the flow itself will not be present in the initial mixture (j = 1).

The reverse effect of the resulting perturbations on the process of combustion will make itself felt through the interaction of the acoustic wave (p'_j, v'_j) and the internal structure of the flame region j = 3, and it will be described by the fieldback equation which we derived [4] with an accuracy to $O(M^2)$:

$$v'_1\big|_{x=-L} - \frac{d\varepsilon}{dt} = qv_1\int\limits_{t-\tau}^{t}\frac{\partial v'_3}{\partial x}\bigg|_{x=v_3(t'-t)}dt',\quad L = v_3\tau,$$

$$q = v_3/v_1 = \alpha - \frac{\alpha-1}{e} > 1,\quad \alpha = v_2/v_1 = \rho_1/\rho_2 > 1. \tag{2}$$

We can bring the perturbed states of regions "1" and "3" into contact at the leading edge x = −L of the flame (in analogy with [5]) by means of the laws of conservation of mass, momentum, and energy on transition through the infinitely small circumference of this edge entirely containing the perturbed position of the flame. Linearization of these laws for x = −L will yield:

$$v'_3 + \frac{p'_3}{\rho_3 v_3}M_3^2 - \frac{v_3}{c_{p_3}}S'_3 = \left(v'_1 + \frac{p'_1}{\rho_1 v_1}M_1^2\right)q,$$

$$2v'_3 + \frac{p'_3}{\rho_3 v_3}(1+M_3^2) - \frac{v_3}{c_{p_3}}S'_3 = 2v'_1 + \frac{p_1}{\rho_1 v_1}(1+M_1^2),$$

$$v'_3 + \frac{p'_3}{\rho_3 v_3} + \frac{v_3}{c_{p_3}}S'_3\frac{1}{(\varkappa_3-1)M_3^2} = \left(v'_1 + \frac{p'_1}{\rho_1 v_1}\right)\frac{1}{q}. \tag{3}$$

We can bring the perturbed states of regions "1" and "2" into contact by means of the laws of continuity for flows of mass, momentum, and energy on transition through the flame, and in our case, in analogy with [6], these laws have the form

$$\alpha\left[v'_1 - \frac{d\varepsilon}{dt} + \frac{p'_1}{\rho_1 v_1}M_1^2\right]_{x=-L} =$$

$$= \left[v'_2 - \frac{d\varepsilon}{dt} + \frac{p'_2}{\rho_2 v_2}M_2^2 - \frac{v_2}{c_{p_2}}S'_2\right]_{x=0},$$

$$\left[2v_1' + (1+M_1^2)\frac{p_1'}{\rho_1 v_1}\right]_{x=-L} =$$

$$= \left[2v_2' + (1+M_2^2)\frac{p_2'}{\rho_2 v_2} - \frac{v_2}{c_{p_2}} S_2'\right]_{x=0},$$

$$\frac{1}{\alpha}\left[v_1' - \frac{d\varepsilon}{dt_l} + \frac{p_1'}{\rho_1 v_1}\right]_{x=-L} =$$

$$= \left[v_2' - \frac{d\varepsilon}{dt} + \frac{p_2'}{\rho_2 v_2} + \frac{v_2}{c_{p_2}}\frac{S_2'}{(\varkappa_2 - 1)M_2^2}\right]_{x=0}. \quad (4)$$

Thus, conditions (2)–(4) for the constants B, A_{11}, A_{22}, A_{31}, A_{32}, D_3, and D_2, which play a role in ε and in solutions (1), lead to a system with the following characteristic determinant to find the eigenvalue of ω:

$$\left[\frac{1}{M_1} - 1 + q\left(1 + \frac{1}{M_3}\right)\right] \times$$

$$\times\left(\frac{1}{\alpha} + 1 + \frac{1}{M_2}\right)(\varphi_1 - \varphi_2)(\alpha - 1)\frac{M_3}{2} +$$

$$+1 - \frac{1}{M_1} - \alpha\left(1 + \frac{1}{M_2}\right) - (\alpha - 1)\left(1 + \frac{1}{\alpha} + \frac{1}{M_2}\right) \times$$

$$\times\left(\frac{1}{1 - M_1} + q\varphi_2\right) = 0,$$

$$\varphi_k = \frac{\exp(\gamma_{3k}L) - \exp(-z/q)}{1 + (-1)^k M_3} M_3, \quad z = \frac{\omega L}{v_1},$$

$$\gamma_{3k}L = \frac{z}{g}\frac{M_3}{(-1)^{k+1} - M^3} \simeq (-1)^{k+1} z\frac{M_3}{q}. \quad (5)$$

At the limit, on changing to an incompressible medium, when M = 0, the perturbations of p' and v' should no longer be functions of x, because the continuity equation takes the form $\partial v_j'/\partial x = 0$. In other words, according to (1), $\gamma \to 0$ as $M \to 0$. Therefore, $zM_3/q \to 0$ as $M_3 \to 0$ and, expanding $\exp \gamma_{3k}L$ into a power series, we can present Eq. (5)—accurate to zM_3^2—in the form

$$\exp\left(-\frac{z}{q}\right) = \frac{(2\alpha - 1)M_1 + M_2}{(\alpha - 1)M_3^2} \sim \frac{1}{M_3}. \quad (6)$$

Hence it follows that Rez < 0 (Reω < 0), because the M numbers for slow combustion are excessively small. In other words, the flame in a gas mixture is stable with respect to one-dimensional perturbations, i.e., the compressibility exerts an extremely intensive (the modulus of Re ω is large) stabilizing effect, leading to Re ω < 0 (as opposed to the indeterminate conclusion of ω = 0 within the framework of the hypothesis of incompressibility [1]).

This result is in agreement with the theory of unstable combustion developed by Landau [1] on the basis of an incompressible medium with respect to two-dimensional perturbations. Indeed, it follows from this theory that ω ~ 1/λ, or the intensity of increasing perturbations with time diminishes as their wavelengths λ increase, so that when λ = ∞ we should expect a transition to stability. It is precisely this fact that has been observed in this study in which we considered the effect of compressibility. Thus, the wavelength of the unstable perturbations, more exactly λ/L, should be bounded from above by an extremely large quantity.

On the other hand, viscous dissipation, which is always present under actual conditions, will stabilize the most intense shortwave perturbations, since these are associated with the greatest gradients across a flow. The relative wavelength λ/L of unstable perturbations must therefore also be bounded from below. Hence, in the experiments we should also expect the appearance of unstable perturbations with a specific value for λ_*/L. This statement is in complete agreement with the data known from the experiments in [7, 8]. The calculation [5] which we carried out on viscous stabilization shows a large magnitude for λ_*/L.

NOTATION

p is the pressure; v is the velocity; ρ is the density; S is the entropy; ϰ is the heat-capacity ratio; c is the speed of sound; c_p is the heat capacity; ω is the eigenvalue; L is the width of the flame front; λ is the wavelength of the flame disturbance for the two-dimensional case; ε is the displacement of the flame front, prime means disturbance, symbols 1, 2, 3 correspond to initial mixture, the products of combustion, and the flame regions; M is the Mach number.

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